St Joseph's Catholic Academy, Matlock

Written Calculation Policy

Rationale

This policy has been designed to show progression in written mathematical methods throughout the school. Our written calculation policy is set out to show:

- The objectives stipulated for all four operations by the National Curriculum
- The calculation methods for each year group
- Relevant vocabulary needed at each stage
- Examples of reasoning activities

Concrete, Pictorial and Abstract

Each method has examples as to what it looks like in the concrete, pictorial and abstract forms. All learners are introduced to a calculation method for the first time using concrete manipulatives. Concrete resources from EYFS to Year 6 include: bead strings, Base 10, Place Value counters, Numicon and Snap cubes. Children will then progress through to a pictorial stage before moving to the abstract. During the pictorial stage, children will be taught to use the bar model. The amount of time needed to progress through each stage is unique to each learner.

Mastering Calculation

The new curriculum has a strong focus on mastery and therefore, if a child is fluent in a method for their year group, they should not be moved onto a different method of calculation or a larger set of numbers Instead, children will be encouraged by their teacher to 'go deeper' within this method. This may involve: using it in different contexts; using and applying it to other learning; using it with missing digits or values; explaining or experimenting with different aspects of it; proving answers with pictures or manipulatives; or explaining what has gone wrong in a calculation. Children must also check their calculations through the use of estimation and inverse operations.

Mathematical Vocabulary

The National Curriculum places great emphasis upon the use of correct mathematical vocabulary and children developing this. Throughout school, children are strongly recommended to use and apply mathematical vocabulary when learning a new method or concept. They will be constantly exposed to this, have it expertly modelled by their teacher and be expected to use it themselves when justifying methods.

Mental Methods

Children should always be encouraged to see if they can work out a calculation mentally before trying a written method. Children will be shown number patterns and relationships between numbers throughout the school. Times tables are introduced and taught in specific year groups: Reception: x2; Year 1: x5, x10; Year 2: x4; Year 3: x3, x6, x8; Year 4: x7, x9, x11, x12.

Key Stage 1 - Year 1

	+	-	x	÷
National Curriculum Objective	• Read, write and interpret statements involving addition, subtraction and equals sign. • Add and subtract 1 and 2 digit numbers to 20, including zero. • Represent and make number bonds and related subtraction facts within 20. • Regroup to 10 to make 10.		• Double and halve numbers to 10 through grouping and sharing. • Make links to counting in multiples of 2, 5 and 10 – drawing arrays. • Reason about odd and even numbers and relate to doubling and halving. • Solve one-step problems involving multiplication and division. • Share objects into equal sized groups.	
Suggested calculation	• Counting on using number lines and number tracks • Informal partitioning	Counting back	Repeated addition and arrays	Sharing and grouping
Mathematical vocabulary	count on, count back, number bonds, number facts, subtraction facts, fact family, add, subtract, more, less, plus, minus, total, sum, difference between, equal		grouping, sharing, multiply, divide, double, half, array, lots of	

Year 2

	+	-	x	÷
National Curriculum Objective	• Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 • Understand the = sign and how 20 + 2 and 24 – 2 both have the same value of 22. • Add and subtract numbers: - a 2-digit number and ones (no regrouping & regrouping in the ones) - a 2-digit number and tens (no regrouping & regrouping in the ones)		• Introduction of arrays in a grid method. • Write mathematical statements using the multiplication (×), division (÷) and equals (=) signs. • Link multiplication and division through missing number questions. • Make links to counting in multiples of 4 and 8. • Share and group objects. • Group using repeated subtraction.	
Suggested calculation	• Informal partitioning • Partitioning column		• Arrays in a grid	• Sharing and grouping in arrays
Mathematical vocabulary	Add, subtract, count on, count back, more, less, plus, minus, total, sum, difference, partition, bridge, round, inverse, number line, number facts, multiple of 10, regroup		Inverse, operation, multiplication table, times table, multiply, multiplication, times, product, repeated addition, lots of, array, divide, division, shared by, halve, double	

Lower Key Stage 2 - Year 3

	+	-	x	÷
National Curriculum Objective	• Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100. • Add and subtract numbers: - up to three digits - two 2 digit numbers - adding 3 one digit numbers		• Multiply 2 digit numbers by a 1 digit. • Introduction of the grid method. • Make links to counting in multiples of 3,6,9. • Share using place value headings TO ÷ O. • Introduce remainders.	
Suggested calculation	Expanded column method		• Grid method	• Sharing and grouping within place value columns
Mathematical vocabulary	Add, subtract, count on, count back, more, less, plus, minus, total, sum, difference, partition, bridge, round, inverse, number facts, multiple of 10, regroup		Inverse, operation, multiplication table, times table, multiply, multiplication, times, product, repeated addition, lots of, array, divide, division, shared by, halve, double	

Year 4

	+	-	x	÷
National Curriculum Objective	Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction.		• Multiply 2/3 digits by a 1-digit number. • Recall multiplication & division facts up to 12 x 12.	Divide numbers up to 3 digits by a 1 digit number using the formal written method of short division and interpret remainders appropriately in context
Suggested calculation	Compact column		• Expanded short multiplication • Compact short multiplication	• Expanded short division
Mathematical vocabulary	addition, subtraction, sum, total, difference, minus, less, plus, altogether, column addition, column subtraction, regroup, operation, estimate, equal, method, inverse		place value, multiply, multiplication, times, product, divide, division, factor, factor pairs, multiplication & division facts, operation, estimate, multiple, shared equally, array	

Upper Key Stage 2 - Year 5

	+	-	x	÷
National Curriculum Objective	Add and subtract whole numbers with more than 4 digits and decimals, using formal written methods of columnar addition and subtraction.		• Multiply numbers up to 4 digits by a 1 or 2-digit number. • Introduction of expanded long multiplication. • Introduction of compact long multiplication.	Divide numbers up to 3 digits by a 1 digit number using the formal written method of short division and interpret remainders appropriately in context
Suggested calculation	Compact column		• Expanded long multiplication • Compact long multiplication	• Compact short division
Mathematical vocabulary	addition, subtraction, sum, total, difference, minus, less, column addition, column subtraction, operation, regroup, inverse, estimate, digit, place holder, rounding, approximate, accuracy		multiply, multiplication, times, product, commutative, short multiplication, long multiplication, multiplication facts, estimate, multiple, remainder	

Year 6

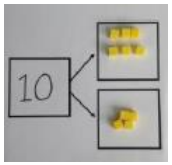
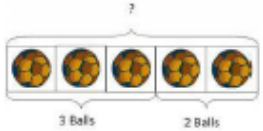
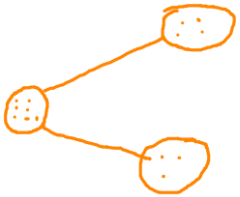
	+	-	x	÷
National Curriculum Objective	Add and subtract whole numbers with more than 4 digits, and decimals with different place values, using formal written methods of columnar addition and subtraction.		• Multiply multi-digit numbers up to 4 digits by a 2 digit whole number using formal written method of long multiplication.	Divide numbers up to 4 digits by a 2 digit number whole number using the formal written method of long division, and interpret remainders as whole number, fractions or decimals
Suggested calculation	Compact column		• Expanded long multiplication • Compact long multiplication	• Long Division
Mathematical vocabulary	addition, subtraction, sum, total, difference, minus, less, column, operation, inverse, estimate, approximate, multiply, multiplication, times, product, commutative, short multiplication, long multiplication, estimate, remainder, fraction, decimal, divisible			

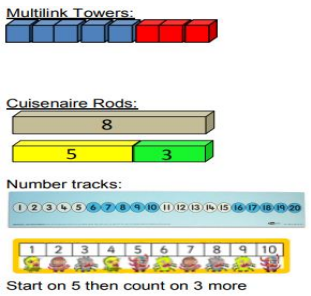
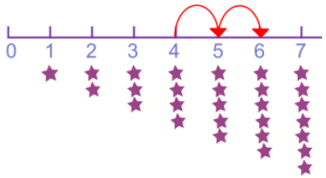
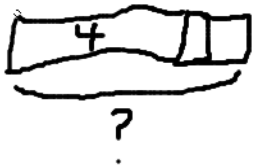
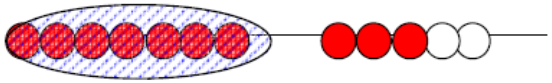
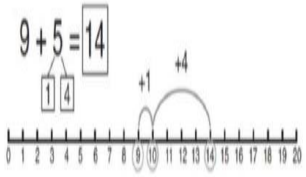
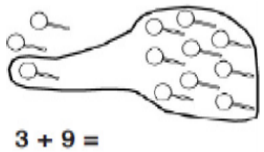
Progression in Written Calculation

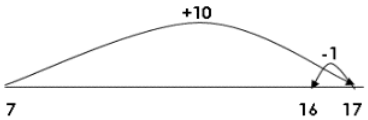
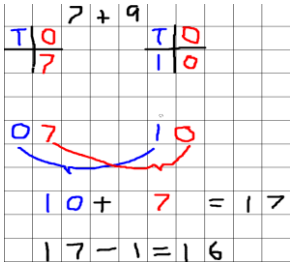
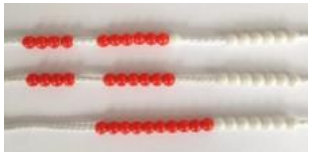
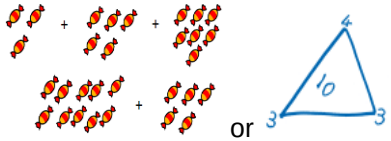
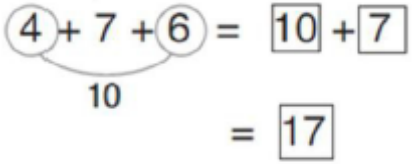
Addition

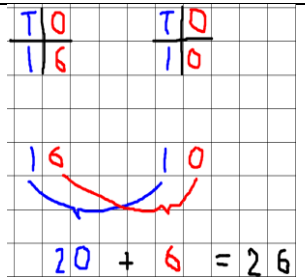
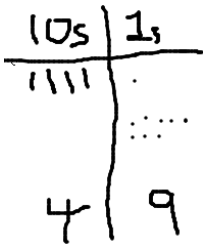
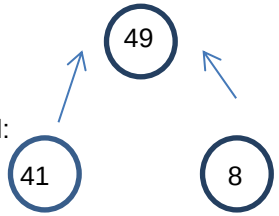
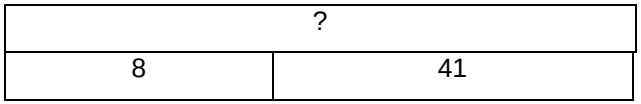
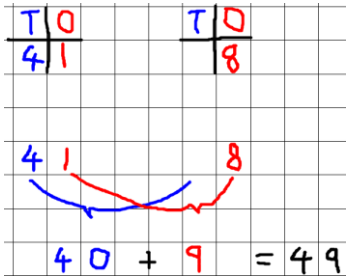
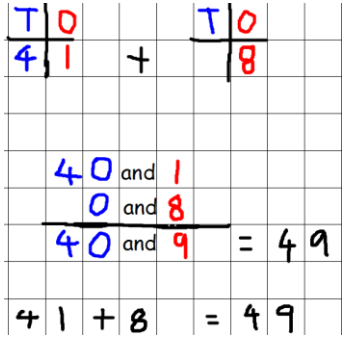
Addition and Subtraction are connected. Addition names the whole in terms of parts, while subtraction names a missing part of the whole.

Part	Part
Whole	

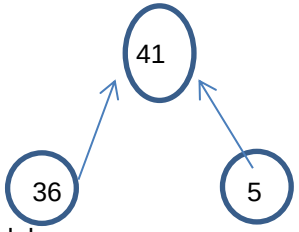
Objective and strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or a bar (aggregation). Start counting at 1. Count one set and then the other. Then count them altogether. (Use other resources – eggs, shells, cars etc)	 Use pictures to add two numbers together. Use the bar model to add two numbers together.  Children draw crosses, dots or numbers in a part-whole model and add together. 	Children start to show recognisable abstract number sentences alongside the pictorial and concrete creations. 4 + 3 = 7 10= 6 + 4 (Equal sign does not have to come at the end).

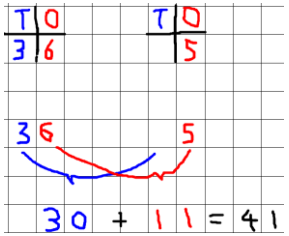
Starting at the bigger number and counting on	**This stage is essential.** Children start to calculate rather than just count. Where one quantity is increased by some amount (augmentation).  Count on from the total of the first set (3 in your head) and count on 2. Always start with the larger number. Use bead strings or Cuisenaire Rods. Number tracks teach children the order of numbers. Number line - points are marked instead (allowing fractions of numbers). Could write number sentence along with creation.	Use a number line with pictorial representation– start at the larger number and count on in ones.  Use a bar model that encourages the children to count on rather than count the whole.  This is an important moment as number lines are very different from number tracks.	Children start to show recognisable abstract number sentences. The sum is 4 + 2 = Or, = 4 + 2 Not, 2 + 4
Regrouping to make 10	Use ten frames and counters/cubes or use Numicon. 6+5 = 11 Use bead strings to show 7 + 5 can be partitioned into 7 + 3 + 2 (children use number bonds to 10). 	Children then draw the ten frame Use a number to partition (decompose) e.g. 9 + 5  Or, use their own pictures to show regrouping 	The sum is 9 + 5 = Children develop an understanding of equality: 6 + ? = 11 6 + 5 = 5 + ? 6 + 5 = ? + 4

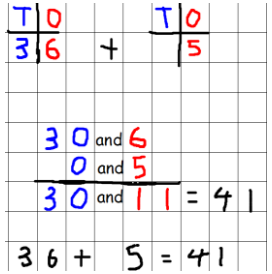
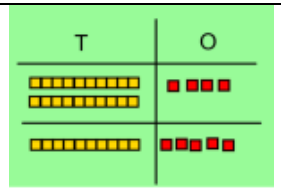
Adding 10 and then compensating	Use a bead string to work out sums e.g. 7 + 9  Children find 7, then add 10 and then adjust by removing 1.	Children draw a picture to show the compensation. 	Introduce the **informal partitioning method:** 
Adding in any order	Explore the commutative law, where you can add in any order, using cubes.	Draw the cubes to show that the total has not changed.	6 + 3 + 4 is the same as 6 + 4 + 3 however the second number sentence is easier & quicker.
Children should be here by the end of year 1			
Year 2			
Adding three single digits	Use bead strings to work out sums e.g. 4 + 7 + 6 = 17. Put 4 and 6 together to make 10. Add on 7.  Build a tower of bricks and then ask the child to split them in 3 ways – add the numbers together.	Add together three groups of objects. Draw a picture to recombine the groups to make 10.  Bar models represent the 3 numbers. ? (10, 20, 30)	Combine the two numbers to make 10 and then add on the remainder.  Children should be shown missing number sentences.
Adding 10	Use cubes or dienes to start with a number and add on 10 more.	Use or draw number squares to count on 10 more by looking at the number directly below.	Children to use **informal partitioning method:**

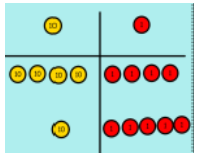
			 Place larger number in your head and add on 10 more by counting in tens. Represent the number sentence in different ways: 41 = 31 +10, 31 + 10 = 41
TO + O **(No regrouping)**	Continue to develop understanding of place value and partitioning e.g. 41 + 8 Using dienes or Cuisenaire rods to show bar models.	Represent base 10 with lines / dots e.g. 41 + 8  The part whole model:  The bar model:  	Children to use **informal partitioning method:**  Introduction of the **partitioning column method:** 

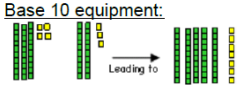
<table>
<tr>
<td>TO + O
(With regrouping in the ones)</td>
<td>Continue to develop understanding of partitioning and place value
36 + 5
Using dienes or Cuisenaire rods to show bar models.</td>
<td>Represent base 10 with lines / dots
e.g. 36 + 5

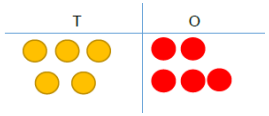
The part whole model:
 

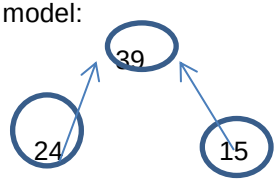
The bar model:
<table>
<tr><td colspan="2">?</td></tr>
<tr><td>36</td><td>5</td></tr>
</table>
</td>
<td>Children to use informal partitioning method:
 

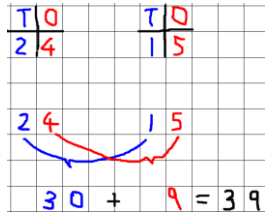
Introduction of the partitioning column method:
 
</td>
</tr>
<tr>
<td>TO + TO
(No regrouping)</td>
<td>Add together the ones first then add the tens.
Use the Base 10 blocks first before moving onto place value counters.
E.g. 24 + 15
 

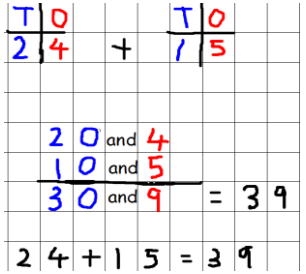
 

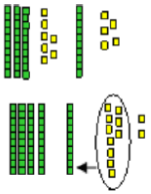
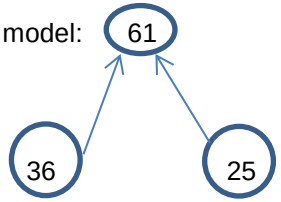
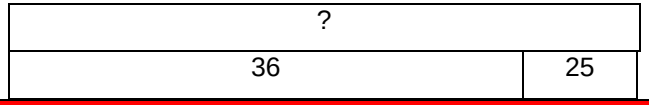
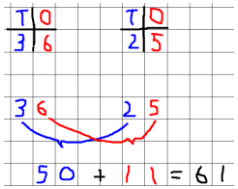
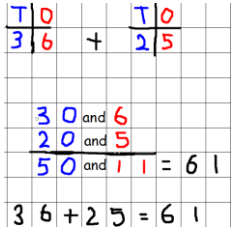
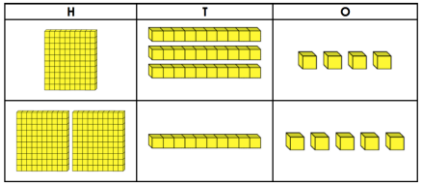
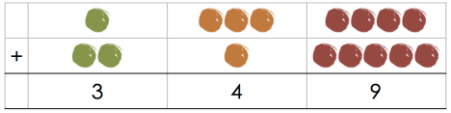
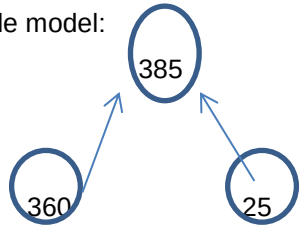
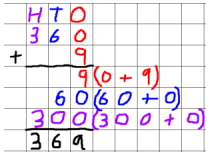
<u>Partitioning (Aggregation model)</u>
34 + 23 = 57
<u>Base 10 equipment:</u>
 

Children create the two sets with Base 10 equipment and then combine; ones with ones, tens with tens.
Using dienes/Cuisenaires to show bar models.</td>
<td>After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.
 

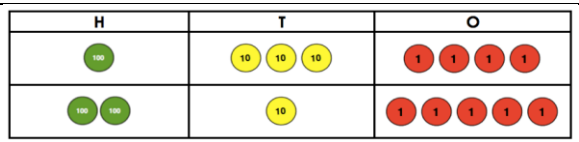
The part whole model:
 

The bar model:
<table>
<tr><td colspan="2">?</td></tr>
<tr><td>24</td><td>15</td></tr>
</table>
</td>
<td>Children to use informal partitioning method:
 

Introduction of partitioning column method:
 
</td>
</tr>
</table>

TO + TO **(With regrouping in the ones)**	Show regrouping using dienes: 36 + 25 37 + 15 = 52  What are the missing counters and digits problems. Use Cuisenaire rods to show bar models.	Children to represent base 10 by drawing it in a place  value chart. The part whole model:  The bar model:  	Children to use informal **partitioning method:**  Introduction of **partitioning column method:** 
Children should be here by the end of Y2			
Year 3			
HTO + O **(No regrouping)** **HTO + O** **(With regrouping)** **HTO + TO** **(No regrouping)** **HTO + TO** **(With regrouping in the tens)**	This can also be done with place value counters or Base 10. 134 + 215 =  	Pictorial representation of the columns and counters.  The part whole model:  	Recap the **partitioning column method.** Introduce the **expanded column method.**  Regrouping:

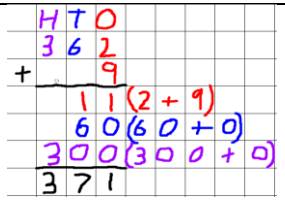
HTO + TO (With regrouping in ones & tens)



Use Cuisenaire rods to show bar models.

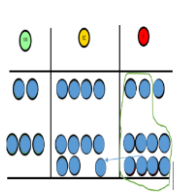
Bar models to show children what to do in a sum.

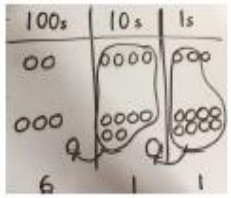
? (sum)	
360 (addend)	25 (addend)

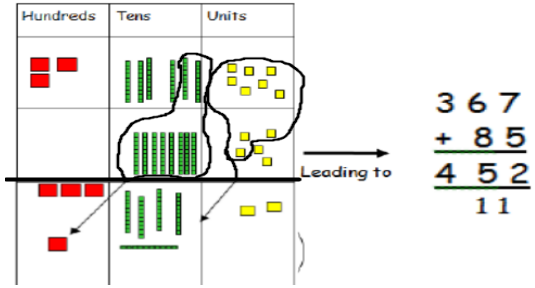


Children should be here by the end of Y3

Y4 – compact column method up to 4 digits and two decimals (introduced with money) with the same number of digits. If there are more than 10 or more counters in a column regroup into a new counter in the next column. Then add up all the columns. Make it, Draw it, Write it.







3 6 7
+ 8 5
4 5 2
1 1

Y5 – compact column method with more than 4 digits and decimals with different number of digits and regrouping in some columns.

Make it, Draw it, Write it.

	£	2	3	.	5	9
+	£		7	.	5	5
	£	3	1	.	1	4
		1	1		1	

Use place value counters for adding decimals

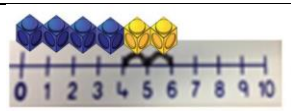
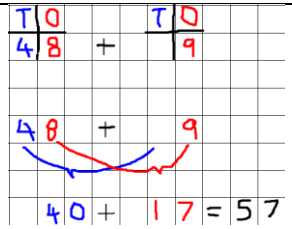
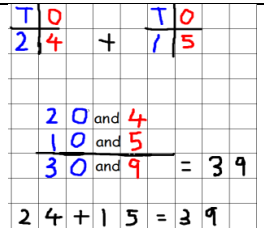
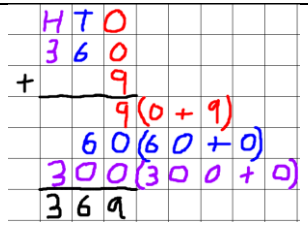
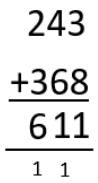
Y6 – compact column method with more than four digits and decimals with different place value and regrouping in some columns.

Make it, Draw it, Write it.

2	3	.	3	6	1	
	9	.	0	8	0	
5	9	.	7	7	0	
+	1	.	3	0	0	
9	3	.	5	1	1	
2	1		2			

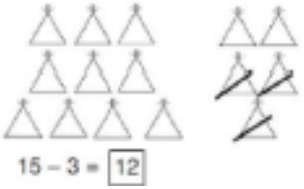
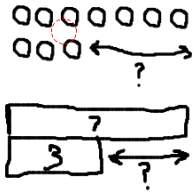
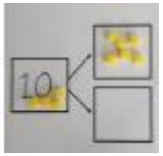
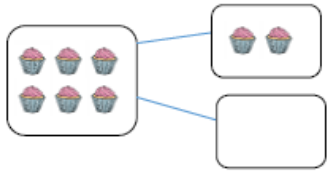
Use place value counters for adding decimals

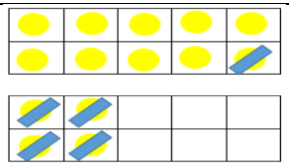
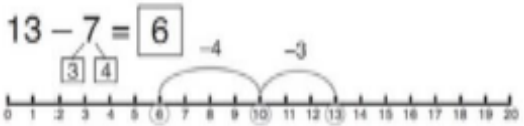
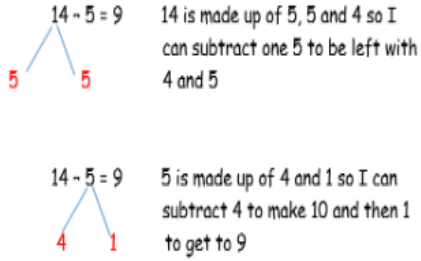
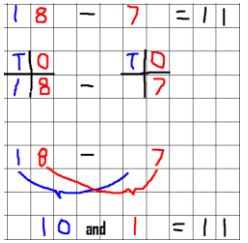
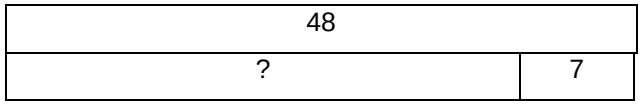
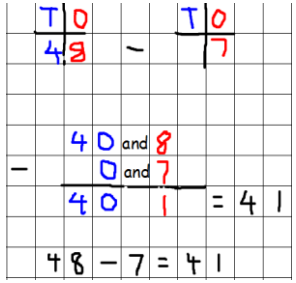
Procedural variation; different ways to ask children to solve 21 + 34

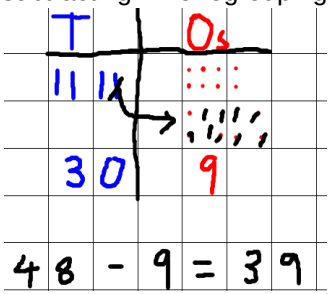
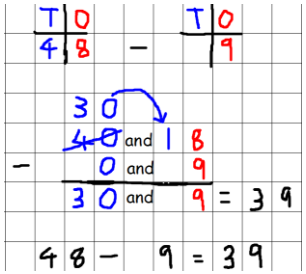
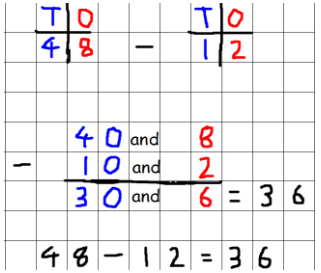
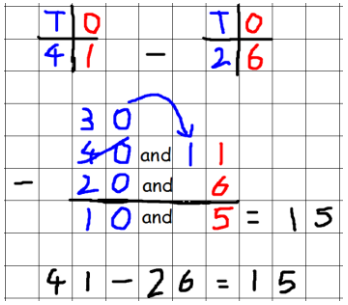
Quick Glance Addition Written Methods:		
Year Group	**Written Method Name**	**Written Method Example**
EYFS	Number tracks and Number lines	
Year 1	Informal Partitioning	
Year 2	Partitioning column	
Year 3	Expanded column	
Year 4	Compact column	
Year 5	Compact column	
Year 6	Compact column	

Quick Glance Addition Number Size (Children must stay within these boundaries)	
Year Group	**Number size**
EYFS	Up to 1 digit + 1 digit
Year 1	Up to 2 digits + 1 digit
Year 2	Up to 2 digits + 2 digits
Year 3	Up to 3 digits (1000)
Year 4	Up to 4 digits including two decimal places
Year 5	More than 4 digits and decimals
Year 6	More than 4 digits and decimals

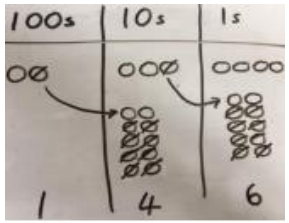
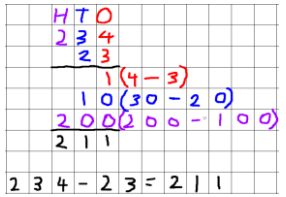
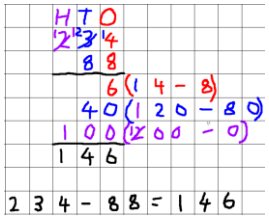
Subtraction

Objective and strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects: counters, cubes etc to show how objects can be taken away (separation model).	Cross out drawn objects to show what has been taken away 	Children start to show recognisable abstract number sentences. 4 – 3 = 1
Counting back	Using number lines or number tracks. Children start with 6 and count back 2	Represent on number line (full and empty).	Children start to show recognisable abstract number sentences. 6 – 2 = 4
Find the difference	Finding the difference using cubes, bead strings Numicon or Cuisenaire rods (comparison model).	Draw the cubes. Use the bar model. 	Find the difference between 8 and 5 8 – 5 = Explore why 9 – 6 = 8 - 5
Part whole model	Link to addition – use the part whole model to help explain the inverse. Explore using counters and bead string. 	Use a pictorial representation of objects 	Use numbers within the part whole model 

Make 10 (bridging 10) by partitioning one of the numbers	14-5 (Numicon, counters, 10 square, bead string)  Take away 4 to make 10 Then takeaway 1 so you have taken away 5. You are left with the answer of 9.	Ten frame:  Number line. Start at 13. Partition the next number. Take away 3 to reach 10. Take away 4. 	Children start to show recognisable abstract number sentences. 13 - 7 = 6, 13 – 6 = 7 
Subtracting 10 and then compensating	18 – 9 Bead string:  Children find 18, then subtract 10 and then adjust by adding 1.	Children draw a picture to show the compensation. Number line: 	18 – 10 = 8, 8 + 1 = 9 Introduce **informal partitioning method:** 
Children should be here by the end of Year 1			
Year 2			
TO – O (No regrouping)	Create the bigger number using base 10/place value counters and then subtract the smaller number. 48 – 7 Children should be advised to use mental methods to calculate this sum initially, before proving their answer with written methods.	Draw the base 10/place value counters and then cross out what you are subtracting. The bar model: 	Introduction of the **partitioning column method:** 

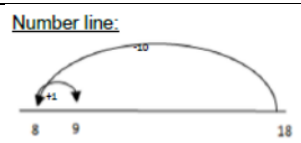
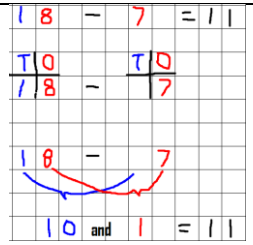
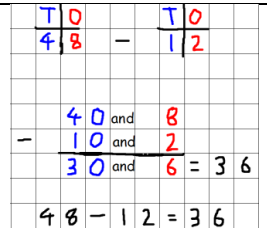
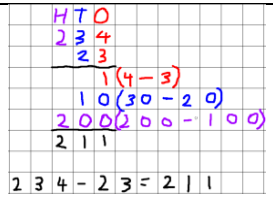
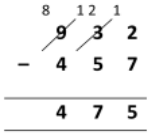
TO – O (With regrouping in the ones)	Create the bigger number using base 10/place value counters and then subtract the smaller number. You can't remove 9 from 8, so you need to 'steal' a ten from the next column. Regroup into 30 and 18. Children can play around with numbers that can add to 48. 48 – 9	Draw the base 10/place value counters and then cross out what you are subtracting. The regrouping must be clearly shown. 	Introduction of the **partitioning column method:** 
TO – TO (No regrouping)	Create the bigger number using base 10/place value counters and then subtract the smaller number. 48 – 12	Draw the base 10/place value counters and then cross out what you are subtracting. The bar model: 48 ? | 12	Introduction of the **partitioning column method:** 
TO – TO (With regrouping in the ones)	Create the bigger number using base 10/place value counters and then subtract the smaller number. 41 - 26	Draw the base 10/place value counters and then cross out what you are subtracting. The regrouping must be clearly shown. 41 - 26	Introduction of the **partitioning column method:** 
Children should be here by the end of Year 2			

Year 3

HTO - O (No regrouping)		Represent the place value counters pictorially; remembering to show what has been regrouped. 234-88  The bar model:	Introduce the **expanded column method.**  With regrouping: 234 – 88 
HTO - TO (No regrouping)			
HTO - TO (With regrouping in the tens)			
HTO - TO (With regrouping in ones & tens)			

Children should be here by the end of Year 3				
Y4 – compact column method with regrouping with up to 4 digits. Make it, Draw it, Write it.		1 2	1 2 3	4
	-		8	8
		1	4	6
	1			
Y5 – compact column method with regrouping with more than 4 digits. Make it, Draw it, Write it.		2	5 6	1 2 3 . 1 0
	-		2	6 . 5
		2	3	6 . 5
Y6 - compact column method with regrouping with more than 4 digits. Place value counters for decimals with different amount of decimal places. Make it, Draw it, Write it.		8 1 2 1 9 3 2 - 4 5 7 4 7 5 Answer: 475		

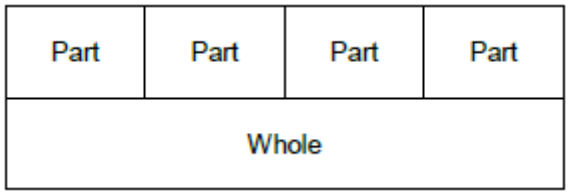
Conceptual variation; different ways to ask children to solve 391 - 186			

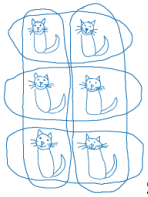
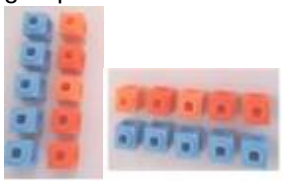
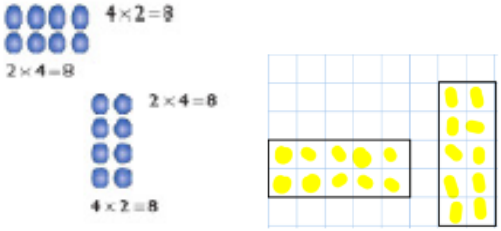
Quick Glance Subtraction Written Methods:		
Year Group	**Written Method Name**	**Written Method Example**
EYFS/Year 1	Number tracks and Number lines	
Year 1	Number lines and Informal Partitioning	
Year 2	Partitioning column	
Year 3	Expanded column	
Year 4	Compact column	 Answer: 475
Year 5	Compact column	
Year 6	Compact column	

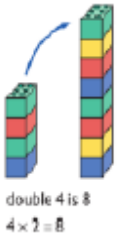
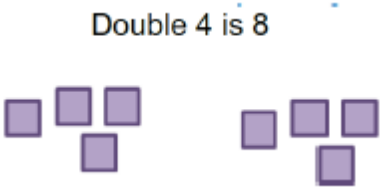
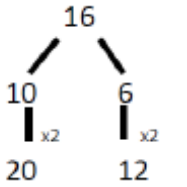
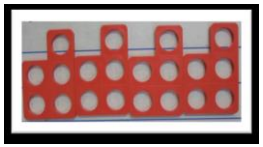
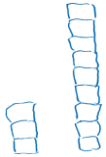
Quick Glance Subtraction Number Size (Children must stay within these boundaries)	
Year Group	**Number size**
EYFS	Up to 1 digit - 1 digit
Year 1	Up to 2 digits - 1 digit
Year 2	Up to 2 digits - 2 digits
Year 3	Up to 3 digits (1000)
Year 4	Up to 4 digits including two decimal places
Year 5	More than 4 digits and decimals
Year 6	More than 4 digits and decimals

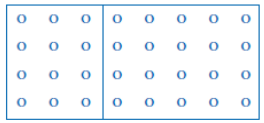
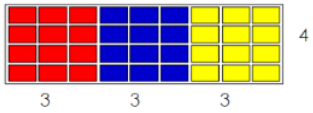
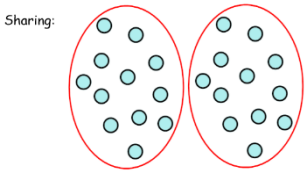
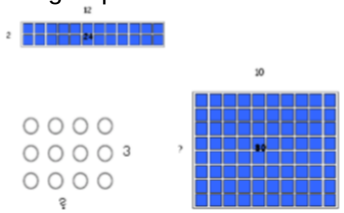
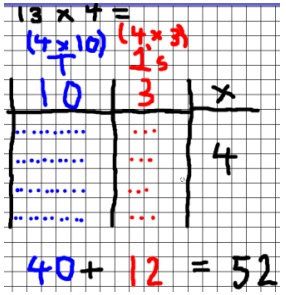
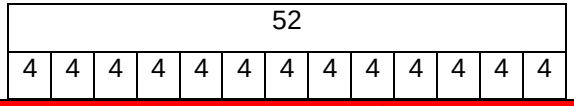
Multiplication

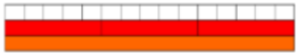
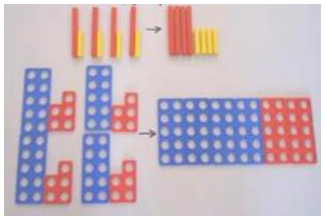
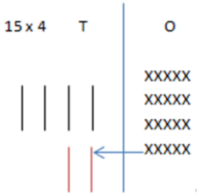
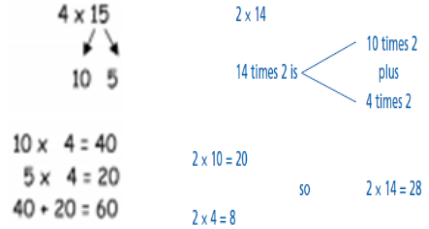
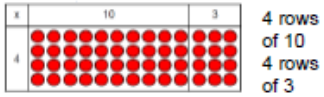
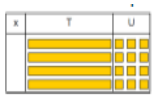
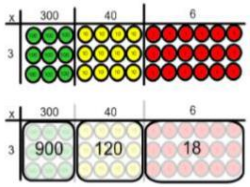
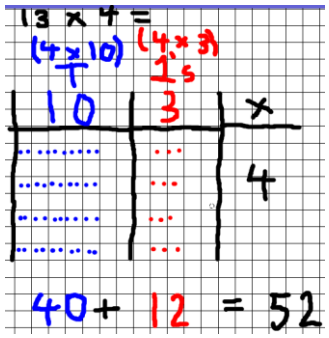
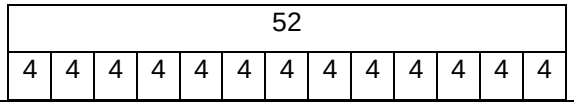
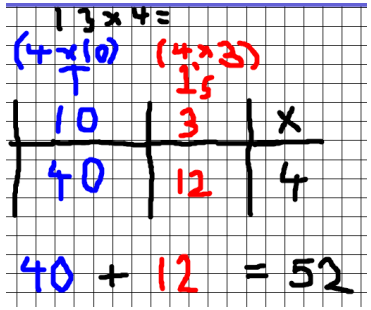
Multiplication and division are connected. Both express the relationship between a number of equal parts and the whole.

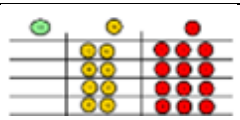
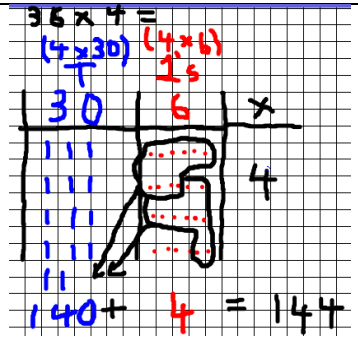
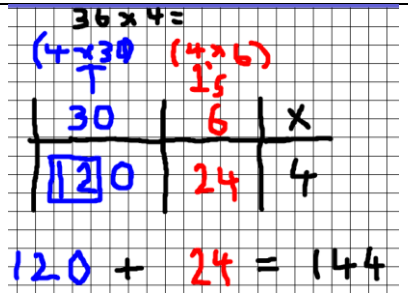
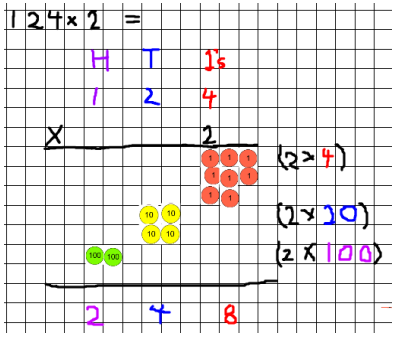
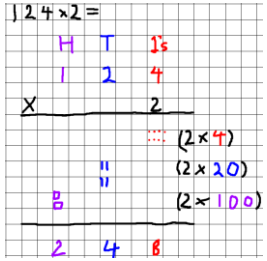
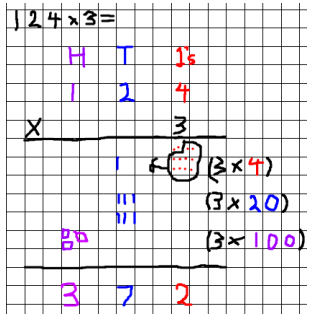
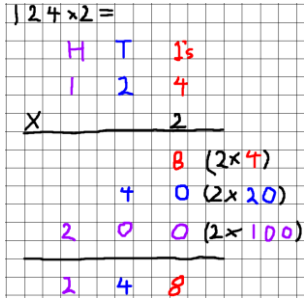
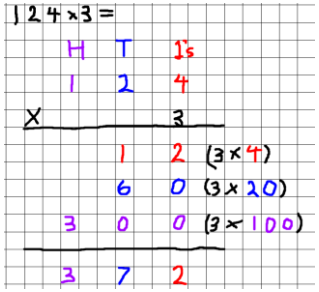


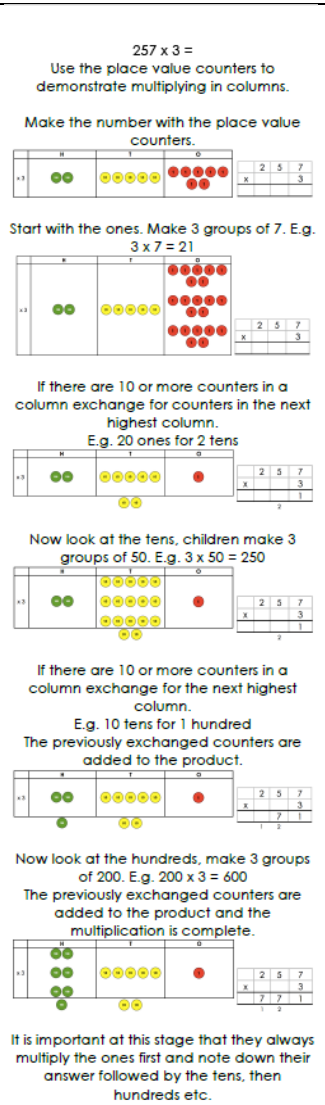
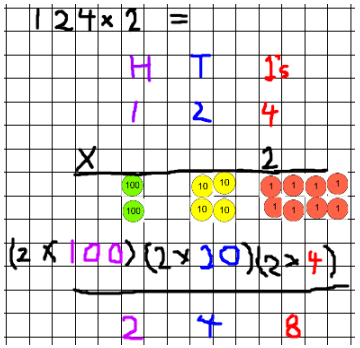
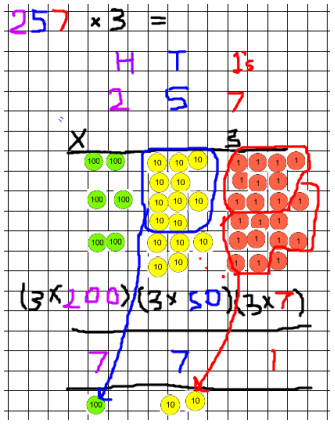
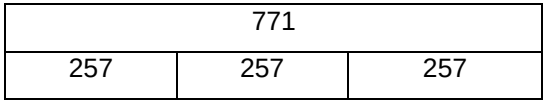
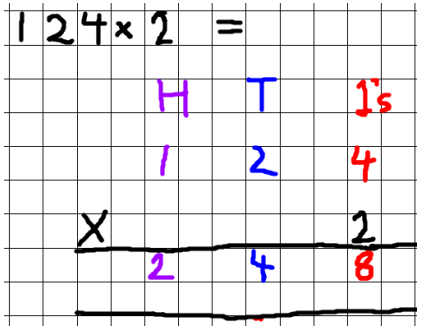
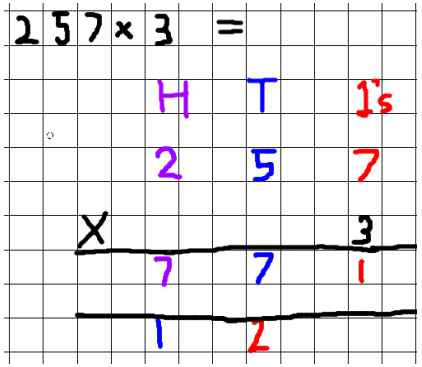
Objectives and strategies	**Concrete**	**Pictorial**	**Abstract**
Multiply by adding equal groups together	Use a set of objects. Double the set by finding the same number again. Make sure both sets are equal.	 Draw the objects  showing: 2 x 3 and 3 x 2	Children may start to show recognisable abstract number sentences. 3 x 4 = 12 4 + 4 + 4 = 12
Introduction of using arrays to count in multiples of 2, 5, 10 (communtative law)	Use a set of objects. Children can place them in groups or start to focus them in on array shapes.  2 x 6 6 x 2	Draw the objects in arrays. Draw in different rotations to find the communtative sentences. This prepares children for the grid method and finding of factors. Also, to help find the area of rectangles. 	Children count in multiples of a number out loud. (See mental mathematics policy for more information). Write sequences with multiples of numbers. 2, 4, 6, 8 etc Children start to use an array to write a range of abstract calculations. 2 x 5 = 10, 5 x 2 = 10, 5 + 5 = 10, 2 + 2 + 2 + 2 + 2 = 10

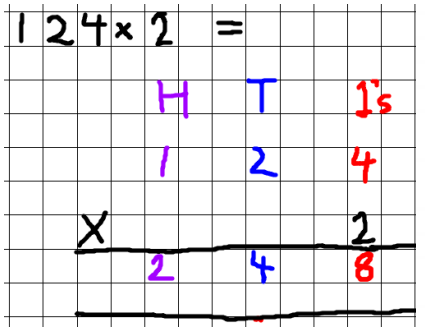
Reason about odd and even numbers and relate to doubling and halving	Create arrays of odd and even numbers with objects – what is the same or different about them? Double the number by adding the same number of objects and discuss what happens.	Draw the objects and circle/highlight the differences and similarities. Draw what happens when you double the number.	Children may start to show abstract number sentences. 3 + 3 = 6 Odd + Odd = Even
Doubling of all numbers up to 10/ **halving**	Use practical activities to show how to double a  number.	Draw pictures to show how to double a number 	 Partition a number, then double each part before recombining
Repeated grouping / repeated addition	There are 3 equal groups with 4 in each group. Use a bead string to show repeated addition.  Children use Cuisenaire Rods to partition totals into equal trains.  Using Numicon to show 4 x 5: 	Make a necklace with red and yellow beads using three red beads for every yellow bead. Use the bricks to make a tower three times as high as this one:  Children represent the practical resources in a picture and use a bar model. Represent on the number line.	Children start to show recognisable abstract number sentences.  Children are taught about the multiplication ‘x’ symbol. 3 x 4 = 12 is the same as 4 + 4 + 4 = 12
Children should be here by the end of Year 1			
Year 2			
Consolidating use of arrays	32 pegs on a board are to be arranged into fours. How can these be shown? This shows the	Ch to illustrate this in different ways and should be encouraged to be flexible with how they use number and	Which could also be seen as 9 x 4 = (3 x 4) + (3 x 4) + (3 x 4) = 12 + 12 +

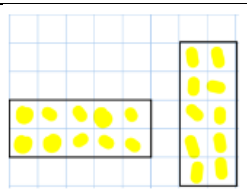
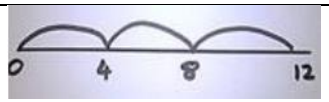
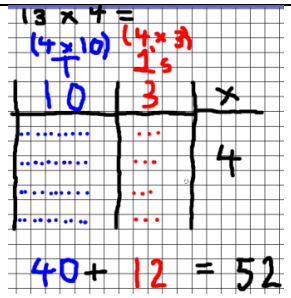
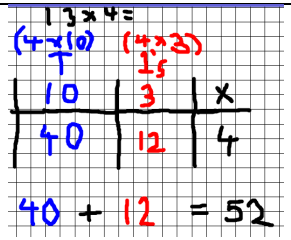
and repeated addition (distributive law)	distributive law where 8 x 4 = 3 x 4 + 5 x 4. 	can be encouraged to break the array into more manageable chunks. 9 x 4 = 	12 = 36 Or 3 x (3 x 4) = 36
Linking multiplication and division through missing number questions	Use objects to make 24. I know there are 2 lots so split them up. How many in each group? 2 x ? = 24 	Drawing arrays or groups: 3 X ? = 12 	Introducing the Inverse operations Trios can be used to model the 4 related multiplication and division facts. 3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3  Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.
TO x O (No regrouping)	Use different resources to create the arrays.	Starting to organise and therefore draw arrays in columns  and show in a bar model. 	
Children should be here by the end of Year 2			
Year 3			
Multiplying factor (scaling)	Increasing a number of objects by a scale factor not by a fixed amount.	Children draw the word problem to find the solution.	Children show recognisable number sentences.

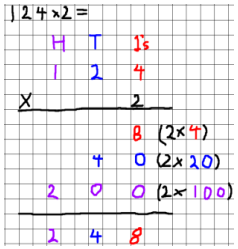
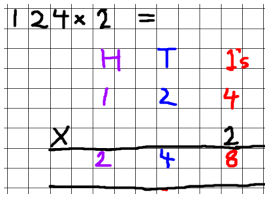
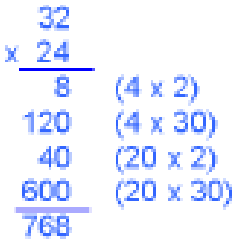
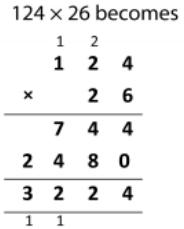
	For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles. This can be written as: 1 + 1 + 1 = 3 □ scaled up by 5 □5 + 5 + 5 = 15 	For example, find a ribbon that is 4 times as long as the blue ribbon.  We should also be aware that if we multiply by a number less than 1, this would correspond to a scaling that reduces the size of the quantity. For example, scaling 3 by a factor of 0.5 would reduce it to 1.5, corresponding to 3 x 0.5 = 1.5.	5 x 4 = 20
Partition to multiply	Use Numicon to show 15 x 4  Show multiplication of the 10s and the 1s separately	Children represent the concrete manipulative in a picture  or	Children show the steps they have taken: 
TO x O (No regrouping)	1) Show the link with arrays with unifix cubes 13 x 4   2) Using Dienes in a grid 3) Using place value counters  	Children can represent the work they have done in a way that they understand.  Show in a bar model.  	Introduction of the **grid method:** 
TO x O (with regrouping of ones into tens)	4 x 23 using place value counters (regrouping). Step 1: Make 4 lots of 23 under place value headings.	Children can represent the work they have done in a way that they understand.	Introduction of the **grid method:** Discuss how multiplying 4 x 3 gives you 12 and the answer is ten times bigger.

	 Step 2: I have 12 counters in the 1's column. Regroup 10 of these into the Ten's column. Step 3: Count the number in each column.		
Children should be here by the end of Y3			
Year 4			
TO x O (no regrouping) **TO x O (extra digit in the answer)** **TO x O (with regrouping of ones into tens)**	Using place value counters to create the sum. 	Children to represent the counters / base 10 pictorially:  With regrouping of 1s into Tens, using Base 10:  Show in a bar model. 372	Introduction of **expanded short multiplication:**  With regrouping of 1s into Tens: 

			124	124	124	
HTO x O (with no regrouping) **HTO x O (with regrouping of ones to tens)** **HTO x O (with regrouping of tens into hundreds)** **HTO x O (with regrouping of ones into tens and tens into hundreds)**		Children to represent the counters / base 10 pictorially:  With regrouping of 1s into Tens (place value counters):  Show in a bar model. 	Introduction of **compact short multiplication:**  With regrouping: 

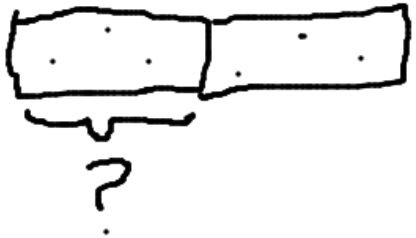
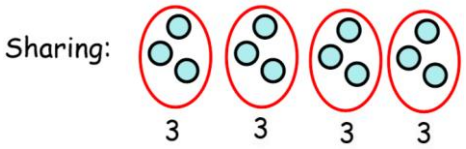
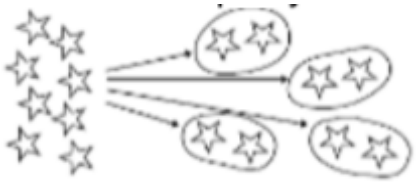
Children should be here by the end of Y4		
Y5 - Introduction of **expanded long multiplication** with up to 4 x 2 digits. Make it, Draw it, Write it. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768	**Y5** - Introduction of **compact long multiplication** with up to 4 x 2 digits**.** Make it, Draw it, Write it. 124 × 26 becomes 1 2 1 2 4 × 2 6 7 4 4 2 4 8 0 3 2 2 4 1 1	**Y6** – Consolidation of **compact short multiplication** and **compact long multiplication** methods with up to 4 digits by a 2 digit. Including multiplication of decimals. Make it, Draw it, Write it.  124 × 26 becomes 1 2 1 2 4 × 2 6 7 4 4 2 4 8 0 3 2 2 4 1 1
Conceptual variation; different ways to ask 6 x 23		

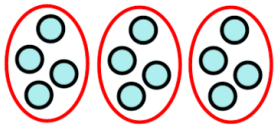
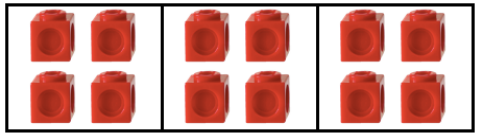
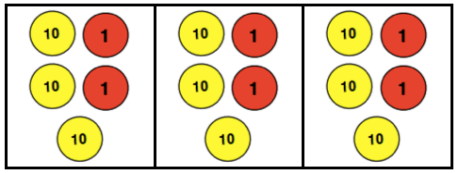
Quick Glance Multiplication Written Methods:		
Year Group	**Written Method Name**	**Written Method Example**
EYFS	Arrays	
Year 1	Arrays and repeated addition	
Year 2	Arrays in a grid	
Year 3	Grid method	

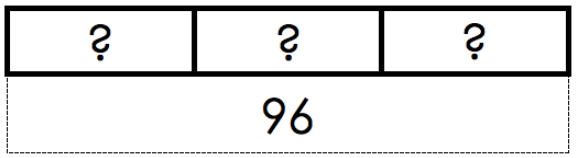
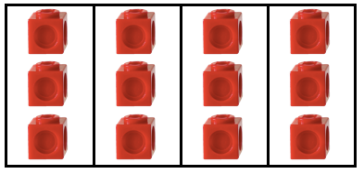
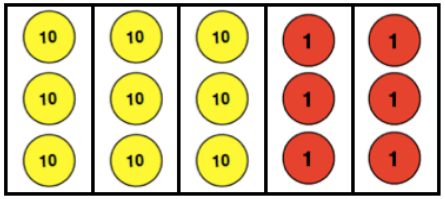
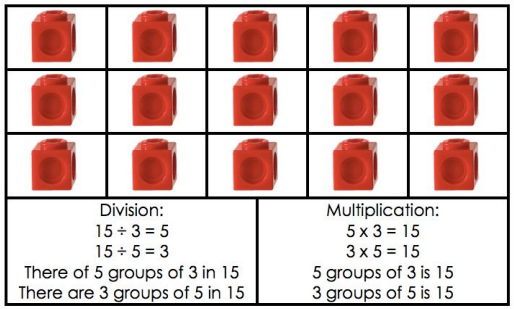
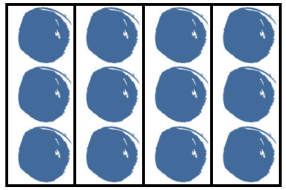
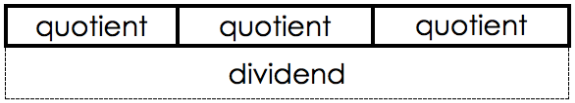
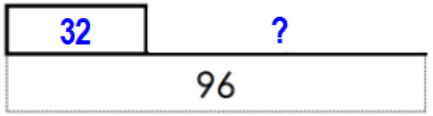
Quick Glance Multiplication Written Methods:		
Year Group	**Written Method Name**	**Written Method Example**
Year 4	Expanded short multiplication and compact short multiplication	**Expanded Short:**  **Compact Short:** 
Year 5 **Year 6**	Expanded long multiplication and compact long multiplication	**Expanded Long:**  **Compact Long:** 

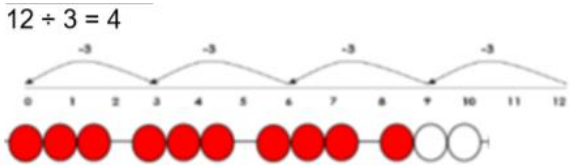
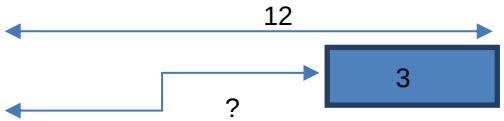
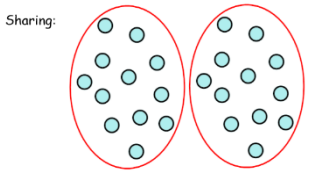
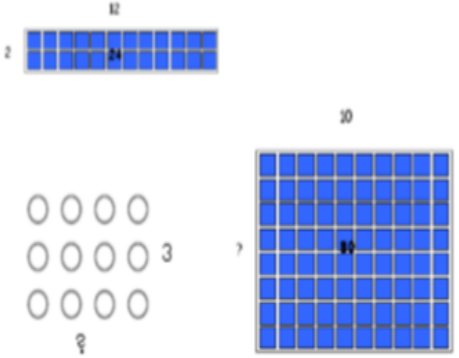
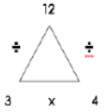
Quick Glance Multiplication Number Size (Children must stay within these boundaries)	
Year Group	**Number size**
EYFS/Year 1	Up to 2 digits x 1 digit
Year 1	Up to 2 digits x 1 digit
Year 2	Up to 2 digits x 1 digit
Year 3	Up to 2 digits x 1 digit
Year 4	Up to 3 digits x 1 digit
Year 5	Up to 4 digits x Up to 2 digits
Year 6	Up to 4 digits x Up to 2 digits

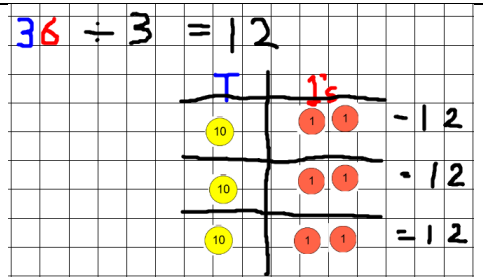
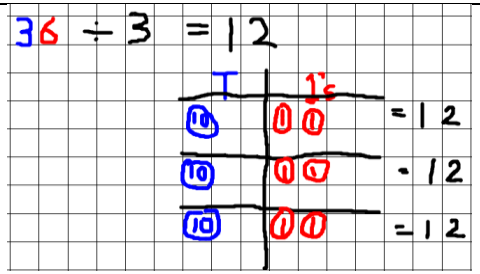
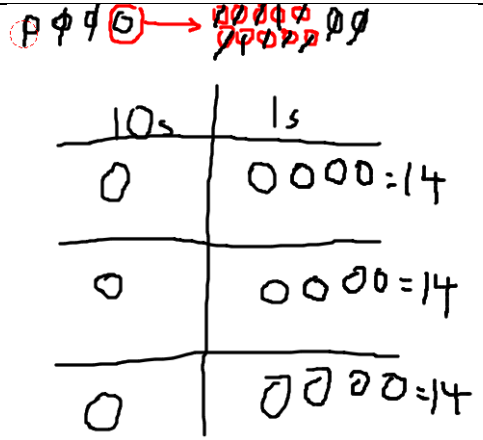
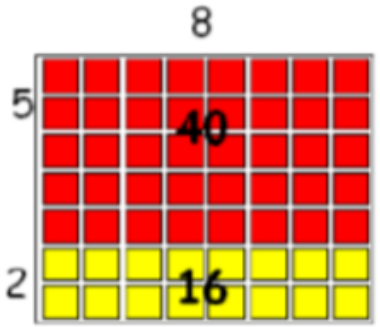
Division

Objectives and strategies	Concrete	Pictorial	Abstract
Equal groups	Children will group different objects into equal sized piles.	Children will draw equal sized groups of objects.	
Sharing objects into equal sized groups	I have 6 cubes; can you share them equally into 2 groups? 	Represent the idea pictorially and using a bar. 	6 ÷ 2 = 3 Children should be encouraged to link these ideas to their times tables facts. Ch could draw bars with abstract numbers in them.
Solve problems which involved sharing or grouping **Know all halves to 10 through grouping and sharing**	Sharing: Introduce practical problems which the children can physically solve. 1) Look at the number that we are dividing e.g. 12 2) Share this number out equally into section of the number that we are dividing by e.g. 4 3) Count how many there are in each section 6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses. How much does each person get? 12 ÷ 4 = 3 	Draw a picture to show what happened. Sharing:  Grouping: 	

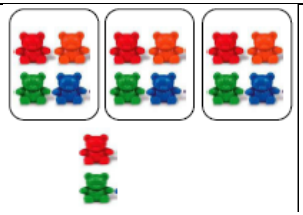
	Grouping: 1) Look at the number that we are dividing e.g. 12 2) Count or draw this many objects 3) How many groups of the number we are dividing by (e.g. take 4 objects and make one group) can you make? 4) Count how many groups you have made There are 6 sweets. How many people can have 2 sweets each? 12 ÷ 4 = 3 Grouping: 		
Children should be here by the end of Y1			
Year 2			
Sharing objects into groups	Share objects into groups. I have 12 cubes.  Can they be shared equally in 3 groups? After sharing between 3 groups we have found that are 4 in each group.  12 ÷ 3 = 4 Using place value counters e.g. 96 ÷ 3 = 32  After sharing we found there were 3 tens and 2	Use pictures or shapes to share quantities.  12 ÷ 3 = 4 Bar Modelling: Split the bar into the number of groups you are dividing by and work out how many would be within each group. Children do not need to use these words! no. of boxes = divisor quotient | quotient | quotient dividend dividend ÷ divisor = quotient e.g. 96 ÷ 3 =	Share 12 sweets between 3 people. 12 ÷ 3 = 4 Share £96 between 3 children. £96 ÷ 3 = £32

	ones in each group.		
Grouping objects	Divide quantities into equal sized groups. I have 12 cubes.  After making groups of 3 we discovered there were 4 of them.  12 ÷ 3 = 4 Using place value counters e.g. 96 ÷ 3 = 32  After making groups of 3, we find there were 3 groups of ten and 2 groups of one. Creating different arrays using cubes. 	Represent using arrays: How many strawberries will each child have if 30 are shared between 5 children? 30 ÷ 6 = 5  30 ÷ 5 = 6 Arrays are really important as they link to the bus stop method!  Bar Modelling: You know how many would be within each group, but need to find out how many groups. no. of boxes = divisor  dividend ÷ quotient = divisor 96 ÷ ? = 32 	Sweets are sold in bags of 3. If I have 12 sweets how many bags would I need? 12 ÷ 3 = 4 There are 96 children sitting in rows of 3. How many rows are there? 96 ÷ 3 = 32
Grouping using	Using Cuisenaire rods above a ruler. Discuss	Represent using a bar model and link to the	Children are introduced to the ÷ sign.

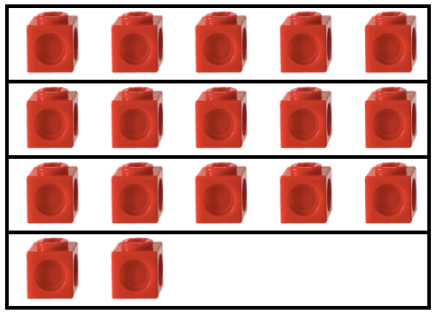
repeated subtraction	that the number sentence (6 ÷ 2 = ?), says, "How many 2s fit into 6?" How big is each hop/rod? Use a bead string to help children to group. 12 ÷ 3 = 4 	Cuisenaire rods and bead strings: 12 ÷ ? = 3  Represent in a Number line to show the equal groups that have been subtracted. The arrows go from the dividend to zero. The number of jumps equals the number of groups.	12 ÷ 4 = 3 12 ÷ 3 = 4 This is linked to the Number line. 12 – 4 – 4 – 4 = 0 12 – 3 – 3 – 3 – 3 = 0 Discuss how division is not commutative e.g. 12 ÷ 3 = 4 but 3 ÷ 12 doesn't = 4 However, 12 ÷ 3 = 4 and 12 ÷ 4 = 3!
Linking multiplication and division through missing number questions	Use objects to make 24. I know there are 2 lots so split them up. How many in each group? 2 x ? = 24 	Drawing arrays or groups: 3 X ? = 12 	Introducing the Inverse operations Trios can be used to model the 4 related multiplication and division facts. 3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3  Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.
Children should be here by the end of Y2			
Year 3			

Sharing **TO ÷ O (with no regrouping and no remainder)**			36 ÷ 3 = 12
Sharing **TO ÷ O (with regrouping and no remainder)**	42 ÷ 3 = 14		Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14
Using the distributive law	Create arrays using cubes. Model grouping the sums into different colours or sections. E.g. 56 ÷ 8 can be done as 40 ÷ 8 and 16 ÷ 8	Ch draw the pictorial representation of the array of  cubes.	Write their sum using abstract numbers. 56 ÷ 8 = 7
Sharing **TO ÷ O**	Use concrete objects to share between groups to see how many is left over. 14 ÷ 3 = 4 r 2	Use a number line to see how many more you need to jump to find a remainder. 13 ÷ 4 = 3 r 1	Complete written divisions and show the remainder using r.

(no regrouping and a remainder)

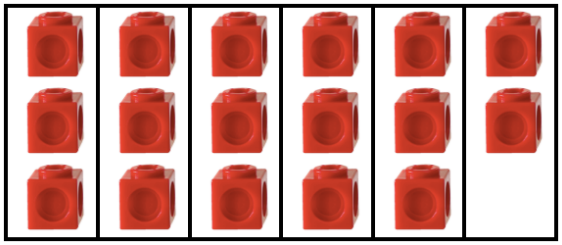


Sharing: 17 ÷ 3 = 5 r 2
Divide objects between groups and see how much is left over.

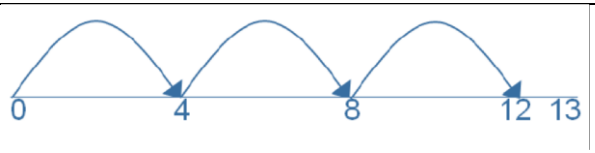


There are 5 in each group with 2 remaining.

Grouping 17 ÷ 3 = 5 r 2
Put objects into groups of 3 and see how many are left over.



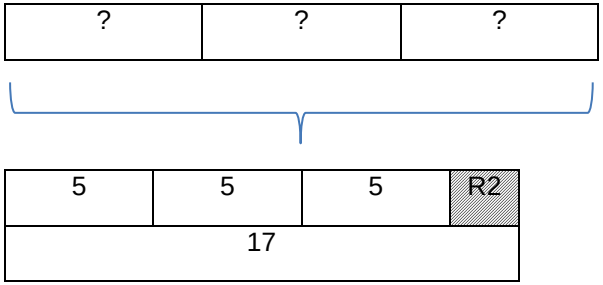
There are 5 groups and with 2 remaining.



Draw dots in circles to divide an amount and clearly show a remainder.
Sharing: 17 ÷ 3 = 5 r 2

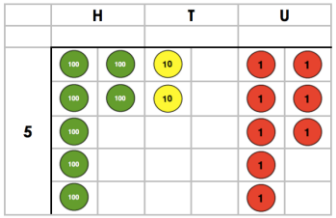
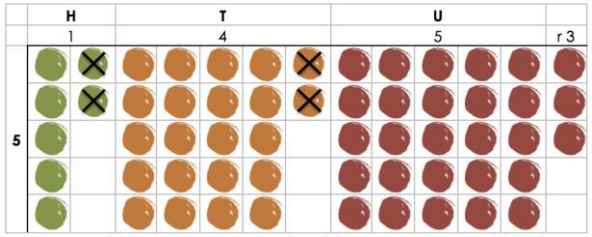
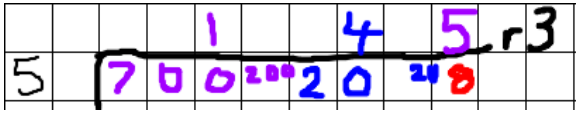
Grouping: 17 ÷ 3 = 5 r 2

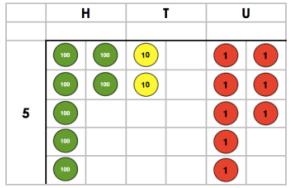
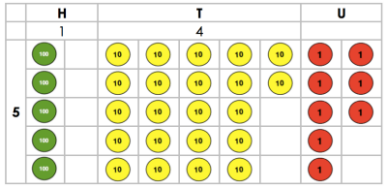
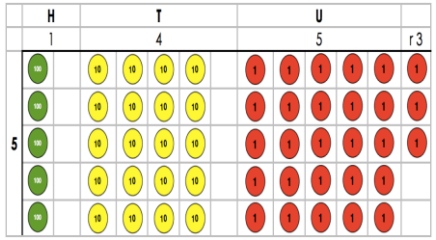
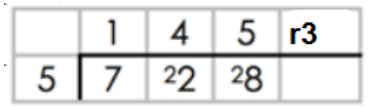
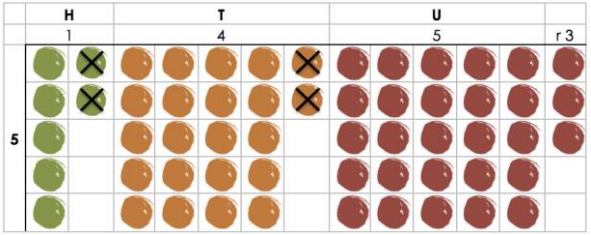
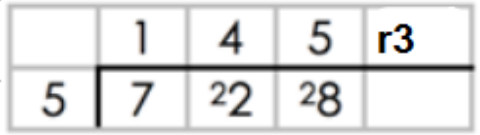
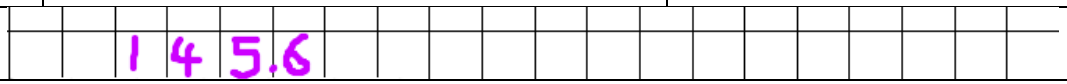
Bar model can be used to display whole being divided into equal parts with a remainder left over.
17 ÷ 3 = 5 r 2

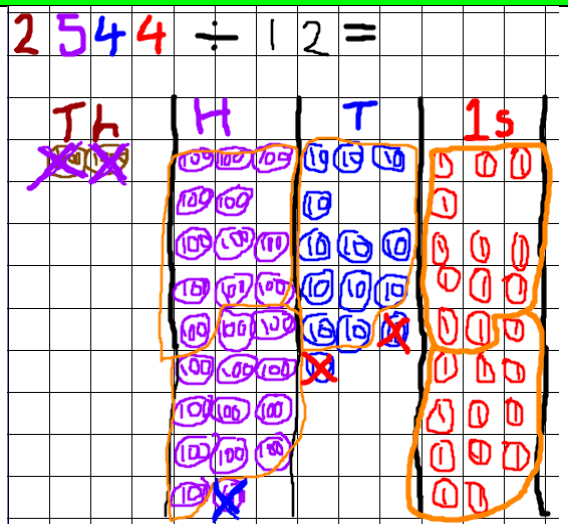


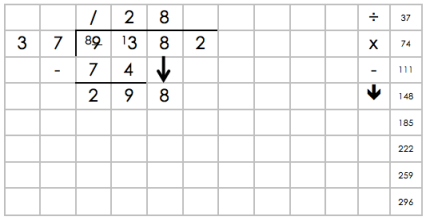
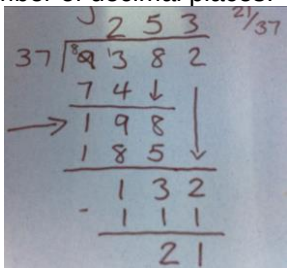
Children should be here by the end of Y3

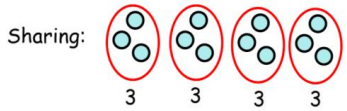
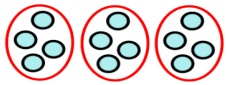
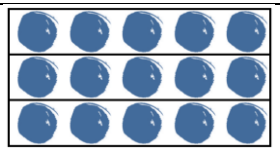
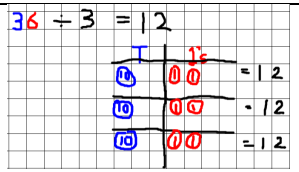
Year 4

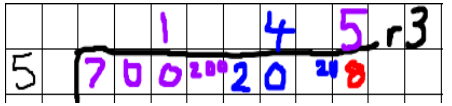
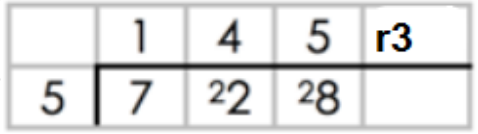
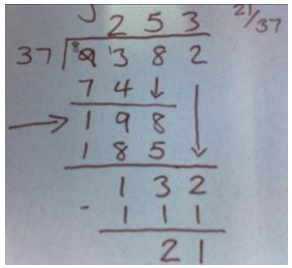
TO ÷ O (No regrouping and no remainder)	Alongside using counters children to see the expanded short division method. 	Draw the counters and cross out any that are regrouped. 	Could introduce the **expanded short division method:**  Discuss how you are trying to find out how many groups of 5 the 700 can be placed in, not how many 5s fit into the number 700. Model alongside the concrete manipulatives and the pictorial representation. Or, use compact short division.
TO ÷ O (No regrouping and a remainder)			
TO ÷ O (Regrouping and no remainder)			
TO ÷ O (Regrouping and a remainder)			
HTO ÷ O (No regrouping and no remainder)			
HTO ÷ O (No regrouping and a remainder)			
HTO ÷ O (Regrouping of hundreds into tens)			
HTO ÷ O (Regrouping of tens into ones)			
HTO ÷ O (Regrouping of hundreds into tens and tens into ones and a remainder)			
Children should be here by the end of Y4			
Year 5			

Where there are zeros in the quotient e.g. 816 ÷ 4=204	728 ÷ 5 =145r3 Layout the bus stop. Place counters in an array with the number of rows reflecting the divisor & keeping to column values.  Start with the biggest place value. 7 hundreds put into groups of 5. 1 group of 5 with 2 hundreds left over. Regroup these 2 hundreds for 20 tens. These are then placed within the array in the tens column.  22 tens put into groups of 5 would be 4 tens with 2 tens remaining. These remaining counters are regrouped once again.  Finally, 28 ones, grouped into 5 would 5 with a remainder of 3.  	Draw a pictoral representation of the columns and place value counters.  Children should only draw in one column at a time. Symbols can be crossed out when regrouping takes place. Bar modelling can support learners when solving problems with division alongside the formal written methods. no. of boxes = divisor quotient quotient quotient quotient quotient dividend dividend ÷ divisor = quotient ? ? ? ? ? 728 728 ÷ 5 = 145 r 3 145 145 145 145 145 r 3 728	Introduction of the **compact short division method:**  Model alongside the concrete manipulatives.
HTO ÷ O no regrouping and no remainder			
HTO ÷ O no regrouping and a remainder			
HTO ÷ O with regrouping of hundreds into tens and no remainder			
HTO ÷ O with regrouping of tens into ones and no remainder			
HTO ÷ O with regrouping and a remainder			
HTO ÷ O with regrouping of tens into ones and a remainder			
HTO ÷ O with regrouping hundreds into tens and a remainder			
HTO ÷ O with regrouping of hundreds into tens and tens into ones and a remainder			
Remainder expressed as a		 	

fraction			
Remainder expressed as a simplified fraction			
Remainder expressed as a decimal			
Children should be here by the end of Y5			
Year 6			
Long division (2 digit divisors)	Long division using place value counters 2544 ÷ 12 We can't group 2 thousands into groups of 12 so we will exchange them into the hundreds column. We can group 24 hundreds int which leaves 1 hundred After exchanging the hundred, we have 14 tens. We can group 14 tens into one group of 12, which leaves 2 tens. After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 groups of 12, leaving no remainder der.		9382 ÷ 37 Encourage children to write the four steps (divide, multiply, subtract and bring down) as checklist. Divide the first 2 digits of the dividend by the divisor. Encourage children to jot the multiples of the divisor on the side to check. In short division the children would be working out the remainder to exchange mentally. Ensure the children understand they are doing the same thing but recording it on the paper, as the number is larger. This is done multiplying the divisor by quotient (E.g. 2 x 37) and then subtracting this from the dividend.

			Rather than writing the remainder in the next column (Short division) the next column is brought down to the remainder.  This process is repeated until each column has been divided leaving a remainder or to a certain number of decimal places. 
Procedural variation – different ways to ask children to solve 615 ÷ 5 =			

Quick Glance Division Written Methods:		
Year Group	**Written Method Name**	**Written Method Example**
EYFS **Year 1**	Sharing and grouping in circles	Sharing:  Grouping: 
Year 2	Sharing and grouping in arrays	
Year 3	Sharing within place value columns	

Year Group	Written Method Name	Written Method Example
Year 4	Expanded short division, or compact short division	**Expanded short division:** 
Year 5	Compact short division	**Compact short division:** 
Year 6	Long division	**Long division:** 

Quick Glance Division Number Size (Children must stay within these boundaries)	
Year Group	**Number size**
EYFS/Year 1	Up to 2 digits ÷ 1 digit
Year 2	Up to 2 digits ÷ 1 digit
Year 3	Up to 2 digits ÷ 1 digit
Year 4	Up to 3 digits ÷ 1 digit
Year 5	Up to 3 digits ÷ 1 digit
Year 6	Up to 4 digits ÷ 2 digit